Maths for Economics

2. The plane, sets, and more notation

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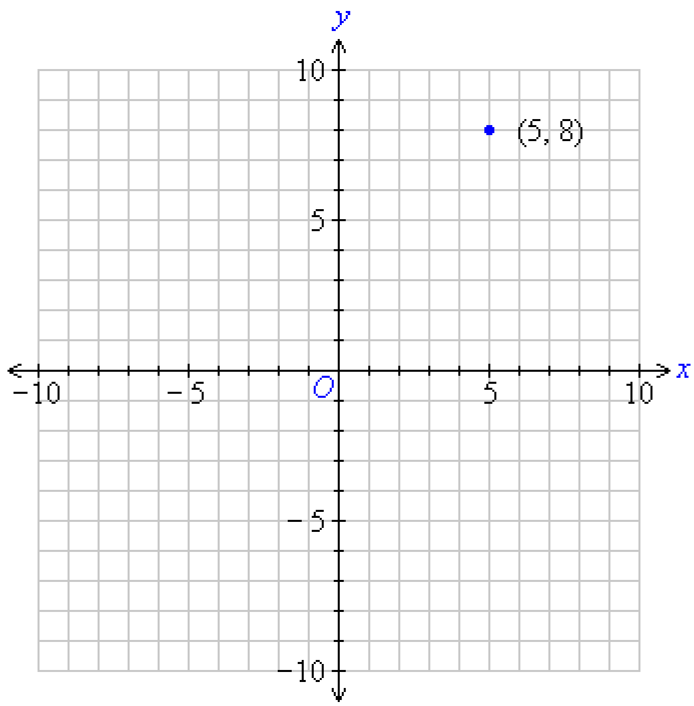
1 The Cartesian plane

Often times we want to talk about pairs of numbers. We can visualize those by combining two number lines into a plane:



We refer to points in the plane with this parenthetical notation: $(5, 8)$ is the point shown in the graph. The first number is the horizontal coordinate, and the second number is the vertical coordinate.

We'll use the Cartesian plane extensively later in the course.

2 Sets

We will also want to be able to describe different sets of numbers. We usually use small letters to represent numbers (as I've done above) and capital letters to represent sets of numbers.

2.1 Other mathematical notation

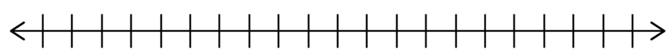
Before we learn how to write down sets of numbers, here are a few useful symbols to know.

- $\in$: "In", or "is an element of". For example: $x \in X$ means that number x is a member of set X.
- $\forall$: "For all". Example: $\forall\ x \in X,\ x > 0$ means that every number x in the set X is positive.
- $\exists$: "There exists". Example: $\exists\ x \in X$ means that there is some number in X (X is not empty).
- s.t., or $|$: "such that". Example: $\exists\ x \in X$ s.t. $x > 0$ means that there is some number in set X that is strictly greater than 0.

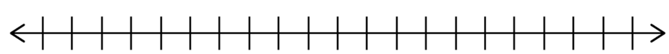
2.2 Set notation

There are a few ways we can describe sets in writing. And since a set of numbers is just a collection of points on the numberline, we can graph them.

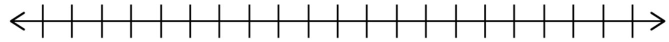
- $\emptyset$: This is the **empty set**. The set that contains no elements is still considered a set, sort of like 0 is still considered a number.
- $[1, 2]$: This is a **closed interval**. It contains all of the numbers between 1 and 2 on the number line, *including* 1 and 2.



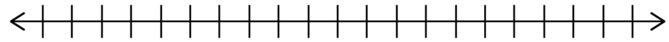
- $(1, 1.5)$: This is an **open interval**. It contains all of the numbers between 1 and 1.5 on the number line, *not including* 1 and 1.5.



- $[\frac{1}{2}, 3)$: This is an interval that is neither open nor closed. It contains all of the numbers between $\frac{1}{2}$ and 3 on the number line, *including* $\frac{1}{2}$ and but *not including* 3.

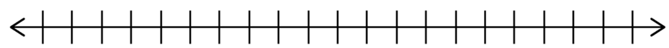


- $(-1, 0]$: This is an interval that is neither open nor closed. It contains all of the numbers between -1 and 0 on the number line, *not including* -1 but *including* 0.

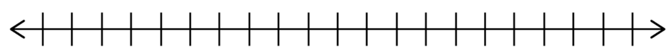


- $(-\infty, -3]$: This is an interval that extends all the way to infinity. Since ∞ itself isn't a number, we *have* to use a round parenthesis when we write down that edge of the interval: Every number between $-\infty$ and -3 is in the interval, including -3, but it *can't* include $-\infty$.

- $\{1, 4, \frac{1}{3}\}$: We use curly braces to define sets other than intervals. This set simply contains three specific numbers.



- $\{x \in \mathbb{N} \mid x > 2\}$: We can also describe the members of a set using more flexible notation and surrounding the description in curly braces. We would read this as "the set of natural numbers x such that $x > 2$".



A more complete compilation of mathematical notation is available at https://en.wikipedia.org/wiki/List_of_mathematical_symbols_by_subject.

2.3 Set manipulation and description

We can also build more complicated sets out of smaller ones, or manipulate them with a new kind of arithmetic.

- $\cup$: This is set addition, or **union**. $X \cup Y$ is a set that combines all of the elements of X and Y. For example, $\{1, 2, 3\} \cup \{2, 4\} = \{1, 2, 3, 4\}$.

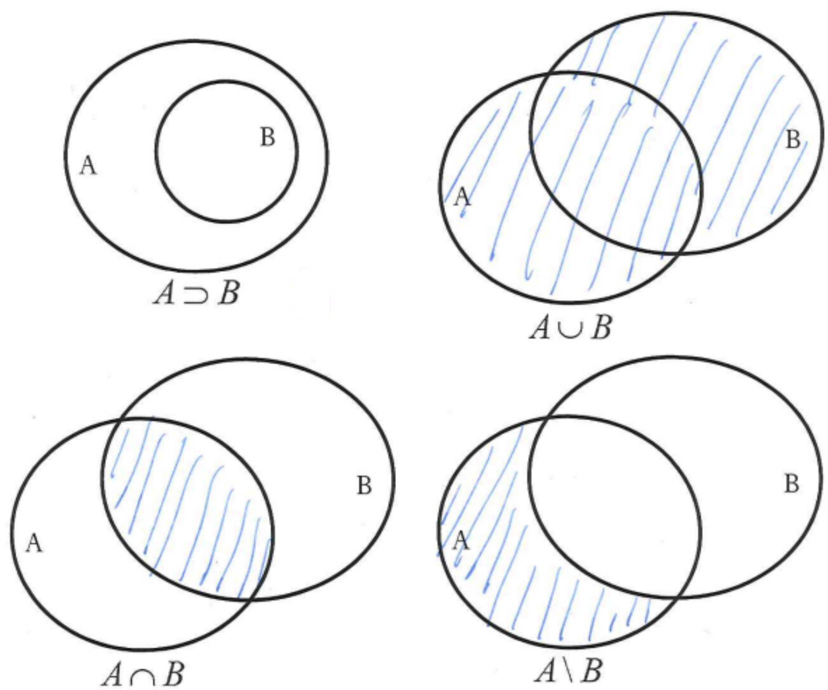
- $\cap$: This is set **intersection**. $X \cap Y$ is a set that contains all of the numbers that are in *both* X and Y. For example, $\{1, 2, 3\} \cap \{2, 4\} = \{2\}$.

- $\setminus$: This is set subtraction. $X \setminus Y$ is the set of numbers in X that are *not* in Y. For example, $\{1, 2, 3\} \setminus \{2, 4\} = \{1, 3\}$.

- $\subset$: $A \subset B$ means that set A is a **strict subset** of B. Every number in A is also in B, and they are not the same set. (That is, there must be some number in B that isn't in A).
- $\subseteq$: $A \subseteq B$ means that set A is a **weak subset** of B. Every number in A is also in B, and they might be identical sets. The word "subset" on its own usually means "weak subset".
- $\supset$: $A \supset B$ means that set A is a **strict superset** of B. That is, $B \subset A$.
- $\supseteq$: $A \supseteq B$ means that set A is a **weak superset** of B. That is, $B \subseteq A$. The word "superset" on its own usually means "weak superset".

2.4 Venn diagrams

Sets don't have to be collections of numbers. They can be groups of any other kinds of objects too: Sets of people, sets of products, set of sets, or anything else. If we want to visualize other kinds of sets than numbers, Venn diagrams can provide a way to reason about them. Imagine a set as a circle that defines the boundary of that collection.



3 Exercises

1. Illustrate the following sets on a number line or plane:

(a) $\{x \in \mathbb{R} \mid x > 2\}$
(b) $X \cap Y$ where $X = [0, 2]$ and $Y = [1, 3]$
(c) $[2, 9) \cap \mathbb{N}$
(d) $[0, 2] \setminus \{x \in \mathbb{Z}\ s.t.\ x/2 \in \mathbb{N}\}$
(e) $\{\frac{1}{2}, \frac{3}{2}\} \cup \{x \in \mathbb{Z} \mid |x| \leq 3\}$

2. List all possible subsets of $\{1, 2, 3\}$.

3. Use Venn diagrams to illustrate and answer the following, with $A = \{1, 2, 3\}$, $B = \{2, 5, 7\}$, and $C = \{1, 2, 8\}$

   (a) $A \cap C$
   (b) $C \cup A$
   (c) $(A \cap B) \setminus C$

4. True or false?

   (a) $A \subseteq B \Leftrightarrow A \cap B = A$
   (b) $A \cup B = A \cup C \Leftrightarrow B = C$
   (c) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$

5. Define $\mathbb{Z}$ and $\mathbb{Q}$ with mathematical and set notation. You can use the symbol $\mathbb{N}$ as well.